Christ Church
Grammar School

**2017**
UNIT TEST 2

# MATHEMATICS METHODS Year 12

**Section One:**
**Calculator-free**

Student name - Solutions -

Teacher name ____________________

## Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet

## *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

**Question 1** **(7 marks)**

Evaluate each of the following integrals (Leave answers with positive indices):

(a) $\int x^4 + \frac{1}{x^2} - \sqrt{x}\ dx$ (2 marks)

$\int x^4 + x^{-2} - x^{\frac{1}{2}}\ dx$

$= \frac{x^5}{5} + \frac{x^{-1}}{-1} - \frac{x^{3/2}}{3/2} + c$ ✓

$\boxed{= \frac{x^5}{5} - \frac{1}{x} - \frac{2x^{3/2}}{3} + c}$ ✓

-1 overall if no '+c'.

(b) $\int \frac{1}{2}\cos\left(\frac{\pi x}{4}\right) dx$ (2 marks)

$= \frac{1}{2}\int \cos\frac{\pi x}{4}\ dx$

$= \frac{\frac{1}{2}\sin\frac{\pi x}{4} + c}{\frac{\pi}{4}}$ ✓

$\boxed{= \frac{2}{\pi}\sin\frac{\pi x}{4} + c}$ ✓

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(c) If $\int_0^k \frac{1}{\sqrt{4x+1}}\, dx = 4$, find the value of k. (3 marks)

$$\int_0^K (4x+1)^{-1/2}\, dx = 4$$

$$\left[\frac{(4x+1)^{1/2}}{\frac{1}{2}\times 4}\right]_0^K = 4$$

$$\left[\frac{1}{2}\sqrt{4x+1}\right]_0^K = 4 \quad ✓$$

$$\frac{1}{2}\sqrt{4K+1} - \frac{1}{2}\sqrt{4(0)+1} = 4 \quad ✓$$

$$\frac{1}{2}\sqrt{4K+1} = 4\tfrac{1}{2}$$

$$\sqrt{4K+1} = 9$$

$$4K+1 = 81$$

$$4K = 80$$

$$K = \underline{20} \quad ✓$$

$$\boxed{\text{If } \tfrac{49}{4}\ \tfrac{1}{3}\text{ mks}}$$

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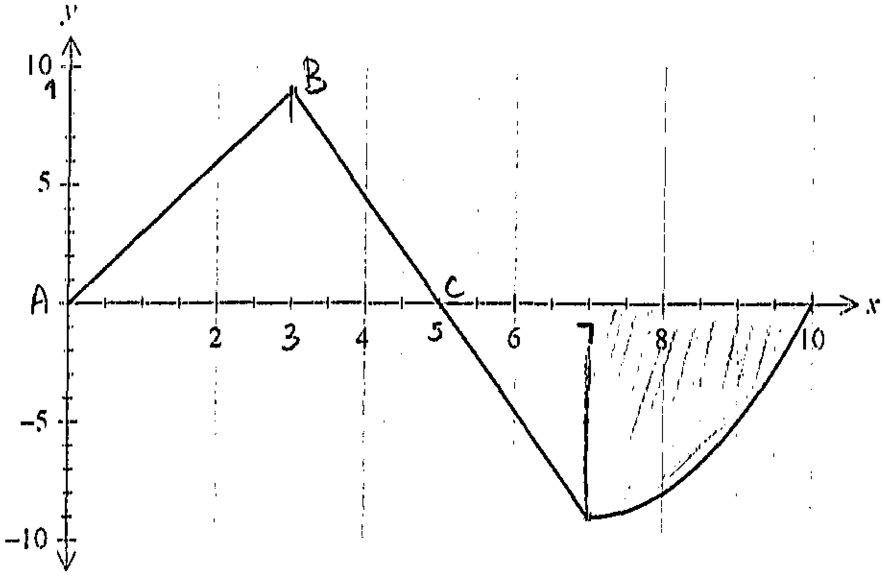
**Question 2** **(4 marks)**

The graph of $y = f(x)$ is shown below. It consists of two straight lines followed by a curve. The area between the function and the $x$-axis is equal to 50 square units.




(a) $\int_0^5 f(x)dx$ (2 marks)

$= \text{Area of } \triangle ABC$

$= \frac{1}{2}(5) \times (9)$ ✓

$= 22\tfrac{1}{2}$ ✓

(b) $\int_7^{10} f(x)dx$ (2 marks)

$= -(\text{Area Shaded})$

$= 50 - (22\tfrac{1}{2} + \tfrac{1}{2}(2)(9))$

$= 50 - (31\tfrac{1}{2})$ ✓

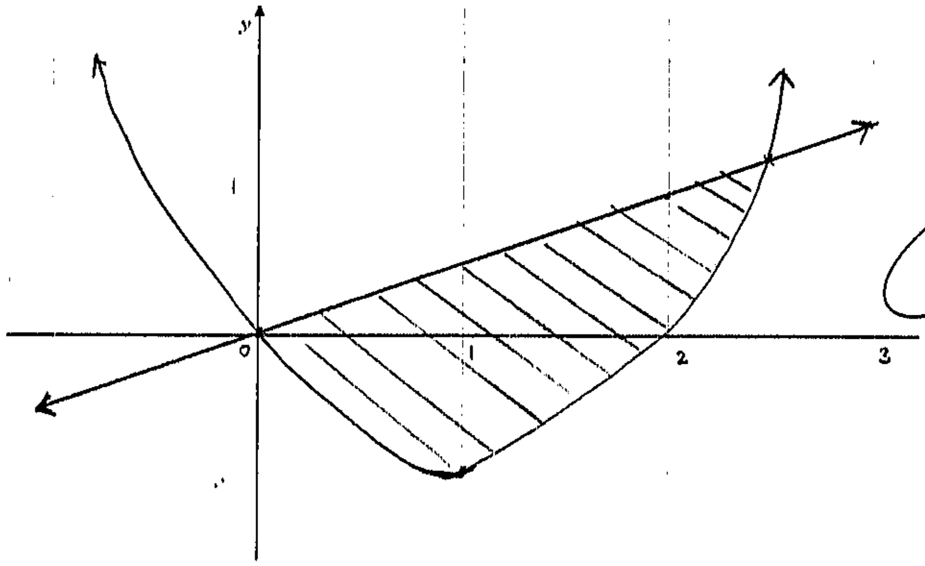
$= 18\tfrac{1}{2}$

$\therefore \int_7^{10} f(x)\,dx = -18\tfrac{1}{2}$ ✓

4

**Question 3** **(4 marks)**

(a) Sketch the curves $f(x) = \frac{x}{2}$ and $g(x) = x^2 - 2x$ on the axes below and shade the area between the curves. (2 marks)




$2x-2=0$

$x^2 - 2x = \frac{x}{2}$

$2x^2 - 4x - x = 0$

$2x^2 - 5x = 0$

$x(2x - 5) = 0$

$x = 0, \frac{5}{2}$

(b) Determine a definite integral that represents the area between the curves.
(There is no need to evaluate the integral) (2 marks)

Area $= \int_0^{5/2} \frac{x}{2} - (x^2 - 2x)\, dx$

OR $A = \int_0^{5/2} \left| \frac{x}{2} - (x^2 - 2x) \right| dx$

End of Questions

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# MATHEMATICS METHODS Year 12

**Section Two:**
**Calculator-assumed**

Student name - Solutions -

Teacher name ______________________

## Time and marks available for this section

Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 30 marks

## Materials required/recommended for this section

***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet (retained from Section One)

## *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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## Instructions to candidates

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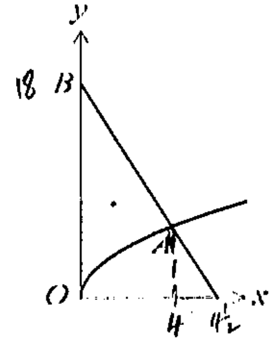
4. It is recommended that **you do not use pencil**, except in diagrams.

**Question 4** **(8 marks)**

The diagram below shows the graph of the function $y = \sqrt{x}$ and the straight line AB that is perpendicular to the curve at A, where $x = 4$.




(a) Determine the equation of AB. (3 marks)

If $y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \Big|_{x=4}$

$= \frac{1}{4}$ ✓

Hence $m_{AB} = -4$ ✓

$\therefore$ Equ $y = -4x + c$ pt $(4,2)$

$2 = -4(4) + c$

$\therefore c = 18$ ✓

Equ $y = -4x + 18$

(b) Determine the shaded area in the diagram, enclosed by the curve $y = \sqrt{x}$, the straight line AB and the y-axis. (2 marks)

$\int_0^4 (-4x+18) - \sqrt{x} \; dx = \frac{104}{3}$ ✓✓

$= 34.\dot{6}$ units$^2$

(c) Determine the area enclosed by the curve $y = \sqrt{x}$, the straight line AB and the x-axis. (3 marks)

AB cuts x-axis at 4.5 $(0 = -4x+18)$

$\therefore \Delta = \frac{1}{2}(4.5) \times 18 - \frac{104}{3}$ from (b) ✓✓

$= 40.5 - \frac{104}{3}$

$= 5.8\dot{3}$ units$^2$ $\left(\frac{35}{6}\right)$ ✓

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**Question 5** **(9 marks)**

A small body is moving in a straight line with velocity $v = 2t^2 - 19t + 30$ m/s, where $t$ is the time, in seconds, since the body first passed through the origin, O.

(a) Determine an expression for $x(t)$, the displacement of the body at time $t$. (2 marks)

$x(t) = \int v(t)\,dt$

$= \int 2t^2 - 19t + 30 \; dt$

$x(t) = \underline{\frac{2}{3}t^3 - \frac{19t^2}{2} + 30t}$ ✓✓ [No '+c' as $t=0$, $x=0$]

(b) Show that the body is stationary twice and find the change in displacement of the body between these two instants. (4 marks)

$\underline{V=0} \Rightarrow 2t^2 - 19t + 30 = 0$ OR $\Delta = 121$

$(x-2)(2x-15) = 0$ $\Delta > 0 \;\therefore 2$ sol$^n$

$x = 2$, $x = 7\frac{1}{2}$ ✓✓

$\int_2^{7\frac{1}{2}} x(t)\;dx = -\frac{1331}{24}$ ✓

$\approx \underline{-55.46\text{ m}}$ ✓

(c) Determine the position of the body when it's velocity is a minimum. (3 marks)

$V'(t) = 4t - 19$ Min $\underline{v'(t) = 0}$

$\underline{4t - 19 = 0}$ ✓

$t = \underline{4.75}$ ✓

$\therefore x(4.75) = -\frac{19}{48}$

$\approx \underline{-0.396\text{m}}$ ✓

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**Question 6** **(4 marks)**

(a) Evaluate the integral $\int_0^2 \left(\frac{1}{1+9x^2} - \frac{1}{10}\right) dx$ to 4 decimal places. (2 marks)

$= 0.2685$ ✓✓

(b) Hence, or otherwise, find the area under the curve of the function $f(x) = \frac{1}{1+9x^2} - \frac{1}{10}$, from $x = 0$ to $x = 2$.

(2 marks)

$\int_0^2 |f(x)|\, dx = 0.3641$ units$^2$ ✓ ✓

**Question 7** **(3 marks)**

A function $f(x)$ passes through the point $\left(\frac{\pi}{6}, -2\right)$. If $f'(x) = \sin(2x)$ find $f(x)$.

$f(x) = -\frac{\cos(2x)}{2} + c$ ✓

Sub in $(\pi/6, -2)$

$-2 = -\frac{\cos \pi/3}{2} + c$

$-2 = -\frac{1/2}{2} + c$ ✓

$\therefore c = -7/4$

$\therefore$ Equ $f(x) = -\frac{\cos(2x)}{2} - \frac{7}{4}$ ✓

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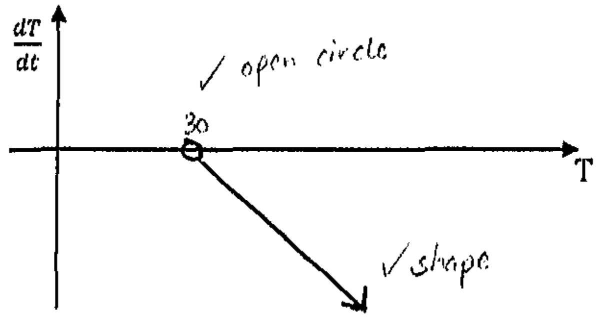
See next page

**Question 8** **(6 marks)**

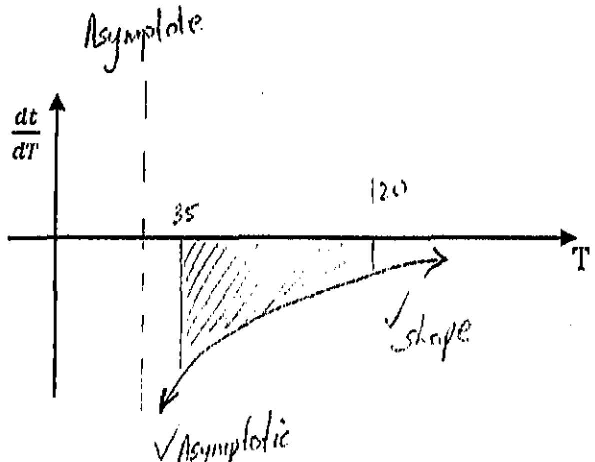
The rate of change of temperature with respect to time of a liquid which has been boiled and then allowed to cool is given by $\frac{dT}{dt} = -0.5(T - 30)$, where $T$ is the temperature (°C) at time $t$ (minutes).

(a) Sketch the graph of $\frac{dT}{dt}$ against $T$ for $T > 30$ below. (2 marks)




| $T$ | 30 | 40 | 50 |
|---|---|---|---|
| $\frac{dT}{dt}$ | DNE | -5 | -10 |

(b) Sketch the graph of $\frac{dt}{dT}$ against $T$ for $T > 30$ below. (2 marks)




| $T$ | 30 | 40 | 50 | 100 |
|---|---|---|---|---|
| $\frac{dt}{dT}$ | DNE | $-\frac{1}{5}$ | $-\frac{1}{10}$ | $-\frac{1}{35}$ |

$$\frac{dt}{dT} = \frac{1}{\frac{dT}{dt}} = \frac{-2}{(T-30)}$$

(c)

(i) Find the area of the region enclosed by the graph of (b), the $x$-axis and the lines $T = 35$ and $T = 120$. Give your answer to two decimal places. (1 mark)

$$\text{Area} = \left| \int_{35}^{120} -\frac{2}{T-30} \, dT \right| = \underline{5.78u^2} ✓$$

(ii) What does this area represent? (1 mark)

Time taken for liquid to cool from 120° to 35°C

✓ MUST HAVE BOTH COMPONENTS

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End of questions